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RESEARCH ARTICLE

Elementary Economic Systems in Material Agent Societies

Sistemas Econômicos Elementares em Sociedades de Agentes Materiais

Antônio Carlos da Rocha Costa[1]*

Abstract: This paper formally characterizes the *elementary economic systems* of material agent societies, on the bases of the notions of (individual and group) *elementary economic behavior*, *elementary economic exchange* and *elementary economic process*. The *equilibrium* of an elementary economic system is defined in terms of the equilibrium of the set of group elementary economic processes that constitute such system. A case study illustrates the proposed concepts.
Keywords: Agent societies — Material agent societies — Elementary economic systems — Elementary economic processes — Elementary economic exchanges — Elementary economic behaviors

Resumo: Este artigo caracteriza formalmente os *sistemas econômicos elementares* das sociedades de agentes materiais, tendo por base as noções de *comportamento econômico elementar*, *troca econômica elementar* e *processo econômico elementar* (tanto individuais como grupais). O *equilíbrio* de um sistema econômico elementar é definido em termos do equilíbrio do conjunto dos processos econômicos elementares grupais que constitui tal sistema. Um estudo de caso ilustra os conceitos propostos.
Palavras-Chave: Sociedade de agentes — Sociedade de agentes materiais — Sistemas econômicos elementares — Processos econômicos elementares – Trocas econômicas elementares — Comportamentos econômicos elementares

[1] *Programa de Pós-Graduação em Computação,Universidade Federal do Rio Grande - FURG, Brazil*
***Corresponding author**: ac.rocha.costa@gmail.com
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1. Introduction

1.1 Contextualization

In a previous paper [1], we have introduced the concepts of *material agent societies* and *energy systems* of material agent societies. The concept of *energy systems* assumes that energy is produced by, and distributed to, the material agents of material agent societies in the form of *energy objects*. The definitions adapt to the realm of material agent societies a reductive reading of some of the main concepts in Hanna Arendt's *Human Condition* [2].

The above two concepts, however, do not determine in any particular way how *producers* and *consumers* of energy objects can interact to produce and distribute such objects. To provide one such way, we focus in the present paper[1] on one particular mode of distribution of energy objects, namely, *elementary economic processes* (akin to Fernand Braudel's *infra-economic processes* [3]). We say that the set of elementary economic processes of a material agent society constitutes the *elementary economic system* of that society.

To allow for elementary economic processes to occur in a material agent society, we require that the material agents be capable of producing other types of objects, besides energy objects, so that different kinds of objects can be exchanged for each other in the *elementary economic exchanges* that constitute those elementary economic processes.

For simplicity, in this paper we consider just one other type of objects, besides energy objects. We call them *chips*. Elementary economic processes are assumed, thus, to involve just the exchange of *energy objects* for *chips*, and vice-versa.

1.2 Structure of the Paper

The paper is structured as follows. Section 2 summarizes the concepts of *material agent*, *material agent society*, *energy system of material agent society*.

Section 3 discusses informally the concepts of *elementary economic behavior* and *elementary economic exchange*.

Sections 4 and 5 respectively introduce formal models for *individual elementary economic behaviors* and *individual elementary economic exchanges*. Section 6, a formal model for *individual elementary economic process*.

Section 7 introduces a formal model for *group elementary economic behaviors and exchange processes*. Section 8, a

[1] This paper is a completely revised and extended version of the paper with the same title presented at the *Workshop-School on Agents, their Environments and Applications* (WESAAC 2018) - see <http://uece.wesaac.com>

formal model for *group elementary economic processes*.

The main subject of the paper, the formal model of *elementary economic system*, is introduced, then, in Section 9.

Section 10 illustrates the application of the formal models introduced in the paper to the *economic modeling and analysis* of some aspects of *ecological systems*.

Section 11 is the Conclusion.

2. Material Agent Societies and their Energy Systems

We first summarize the concepts of *material agent*, *material agent society* and *energy systems* of material agent societies as these concepts were introduced in [1].

We say that an agent is a *material agent* whenever that agent has a *material body*, that is, a body that requires *energy* for its operation. We call *material agent society* any agent society whose agents are all material agents.

We consider here only material agent societies organized around an *energy system*, i.e., a particular social subsystem capable of *producing* and *distributing* energy objects within the society, but in such a way that it guarantees that the society is *energetically autonomous*[2].

We call *energy producer* any material agent that participates in the operation of that energy system. As in [1], we assume that all producers are *energetically self-sufficient*, that is, are capable of producing all the energy they need for their own operation. The other material agents of the society, are said to be *energy consumers*.

3. Elementary Economic Behaviors and Exchanges

We take George Homans' model of *social behaviors and exchanges* [4] as the operational model on the basis of which we define *elementary economic behaviors and exchanges*. This way, our *elementary economical model* builds on the assumption that any material agent *mag* is capable of performing the following two types of actions:

- *deliver* an object to another agent at a time t, denoted by $deliver^t(mag, obj, mag')$, where *mag* is the deliverer, and mag' is the receiver, of the object *obj*;
- *receive* an object from another agent at a time t, denoted by $receive^t(mag, obj, mag')$, where *mag* is the receiver, and mag' is the deliverer, of the object *obj*.

Besides, we assume that each material agent is capable, for each type of existent object, to account for the *sum total of objects* of that type that it has sent, received, or consumed, at each time.

Homans' model is heavily based on Burrhus Skinner's notion of *operant conditioning* [5]. Thus, a central feature of our model is a *reinforcement process* of *sequences* of actions of delivering objects to another material agent (*deliver* actions) by *sequences* of actions of receiving objects of other types from that material agent in return (*receive* actions).

More precisely, *reinforcement processes* are taken to operate between the *temporal rates* of those two sequences, that is, between the *temporal rate* at which the delivering operations are performed, and the *temporal rate* at which the receiving operations are performed.

In consequence, an operational feature of material agent societies that is required for the proper functioning of such reinforcement processes is that the *elementary economic processes* of those societies be *cyclic*, that is, that they be *persistent processes* endowed with a *periodic structure*.

4. Individual Elementary Economic Behaviors

In this section, we sketch a formal theory of *individual* elementary economic behaviors of material agents. We leave for Sect. 8 the extension of the theory to elementary economic behaviors of *groups* of material agents.

4.1 Terms for Denoting Individual Elementary Economic Behaviors

The following *variables* range over the following sets (variables may appear in expressions with various types of decorations):

- *mag*, ranging over the set **Mag** of material agents;
- *beh*, ranging over the set **Beh** of behaviors of material agents;
- *afval*, ranging over the set **AfVal** of affective values of material agents;
- *exch*, ranging over the set **Exch** of exchanges between material agents ;
- *obj*, ranging over the set **Obj** of objects that may be exchanged by material agents;
- $p, q, \ldots$, ranging over the set **Sit** of social situations in a material agent society;
- t and τ, ranging over the linearly ordered set $\mathrm{T} = \{0, 1, \ldots\}$ of time instants.

Behaviors and affective values may be assigned to definite material agents:

- $mag.beh$: a behavior of the definite material agent *mag*;
- $mag.afval$: an affective value of the definite material agent *mag*.

[2] A material agent society is said to be *energetically self-sufficient* whenever the society is organized in a way that allows it to produce all the energy objects its material agents need to perform all the *social functions* that were assigned to them in that society. It is said to be *energetically autonomous* if it is energetically self-sufficient and it can coordinate by itself the production and distribution of its energy objects - see [1].

The *rate of performance* of a behavior *beh* is denoted by the operator "$\langle\ \rangle$" applied to that behavior: $\langle beh \rangle$.

4.2 Basic formulas

Basic formulas have one of the following forms:

(a) statements about *rates of performances* of behaviors, or about *tendencies of variation* in such rates, thus:

- $\langle beh \rangle$: behavior has a defined rate of performance;
- $\langle beh \rangle\top$: behavior has a *high* rate of performance;
- $\langle beh \rangle\boxminus$: behavior has a *medium* rate of performance;
- $\langle beh \rangle\bot$: behavior has a *low* rate of performance;
- $\langle beh \rangle\uparrow$: behavior has an *increasing rate* of performance;
- $\langle beh \rangle\downarrow$: behavior has a *decreasing rate* of performance;

(b) statements about *affective assessments* of rates of performances of behaviors, in the form:

$\langle mag_1.beh \rangle_p [mag_2.afval]_q$

whose meaning is that the *rate* of the behavior *beh* of the material agent mag_1 has a *defined value*, in situation p, and that such rate of behavior is evaluated with affective value *afval* by the material agent mag_2, in situation q. The possible values for affective values *afval* are: $\{-,0,+\}$, each with its intuitive reading.

If $p = q$, one may write: $\langle mag.beh \rangle [mag.afval]_p$. All the components of the formula are optional, except for the $\langle beh \rangle$ component.

In particular, $\langle beh \rangle [\ ]$, the formula just says that the rate of performance of the behavior *beh* of an unspecified material agent, performed in an unspecified situation, is evaluated in an unspecified way, also in an unspecified situation, by some unspecified material agent.

The following illustrate some of the formal variations of such formulas:

- definiteness of the rate of a behavior :
  - any formula in the set:

  $$\{\langle beh \rangle\} \cup \{\langle beh_i \rangle\}_{i \in \mathbb{N}}$$

  meaning that *beh* (or beh_i) has a defined rate of performance;

- situated definiteness of the rate of a behavior:
  - any formula in the set:

  $$\{\langle beh \rangle_p\} \cup \{\langle beh_i \rangle_p\}_{i,p \in \mathbb{N}}$$

  meaning that *beh* (or beh_i) has a defined rate of performance in situation p;

- definiteness of the rate of a behavior of a particular material agent:
  - any formula in the set:

  $$\{\langle mag_i.beh_j \rangle\}_{i,j \in \mathbb{N}}$$

  - the meaning of $\langle mag_i.beh_j \rangle$ is that the rate of the behavior beh_j of material agent mag_i has a defined value;

- situated definiteness of the rate of a behavior of a particular material agent:
  - any formula in the set $\{\langle mag_i.beh_j \rangle_p\}_{i,n,p \in \mathbb{N}}$, meaning that beh_j of mag_i has a defined rate of performance in situation p;

- affective evaluation of the rate of a behavior:
  - any formula in the set:

  $$\{\langle beh \rangle[afval]\} \cup \{\langle beh_i \rangle[afval_j]\}_{i,j \in \mathbb{N}}$$

  - the meaning of $\langle beh \rangle[afval]$ (or $\langle beh_i \rangle[afval_j]$) is that the defined rate of behavior *beh* (or beh_i) is affectively evaluated with value *afval* (or $afval_j$) by an unspecified material agent, in an unspecified situation;

- situated affective evaluation of the rate of a behavior:
  - any formula in the set:

  $$\{\langle beh \rangle[afval]_p\} \cup \{\langle beh_i \rangle[afval_j]_p\}_{p \in \mathbb{N}}$$

  - the meaning of $\langle beh \rangle[afval]_p$ (or $\langle beh_i \rangle[afval_j]_p$) is that the defined rate of behavior *beh* (or beh_i) is evaluated with affective value *afval* (or $afval_j$) by an unspecified material agent, in situation p;

- affective evaluation of the rate of a behavior of a particular material agent by an unspecified material agent:
  - any formula in the set:

  $$\{\langle mag.beh \rangle[afval]\} \cup \{\langle mag_i.beh_j \rangle[afval_k]\}_{i,j,k \in \mathbb{N}}$$

  - meaning that $\langle mag.beh \rangle$ (or $\langle mag_i.beh_j \rangle$) has a defined rate of performance, and is evaluated with affective value *afval* (or $afval_k$) by an unspecified material agent;

- affective evaluation of the rate of a behavior by a specified material agent:
  - any formula in the set:

  $$\{\langle beh \rangle[mag.afval]\} \cup \{\langle beh_i \rangle[mag_j.afval_k]\}_{i,j,k \in \mathbb{N}}$$

  - meaning that the rate of *beh* (or beh_i) has a defined value and is evaluated by material agent *mag* (or mag_j) with affective value *afval* (or $afval_k$);

- situated affective evaluation of the rate of a behavior by a specified material agent:
  - any formula in the set:

  $$\{\langle beh \rangle[mag.afval]_p\} \cup \{\langle beh_i \rangle[mag_j.afval_k]_p\}_{i,j,k,p \in \mathbb{N}}$$

  - meaning that the rate of *beh* (or beh_i) has a defined value and is evaluated by material agent *mag* (or mag_j) with affective value *afval* (or $afval_k$), in situation p.

4.3 Compound formulas

There are three ways of composing formulas: **(a)** *propositional* compositions of formulas; **(b)** *functional dependence* compositions of formulas; and **(c)** *partial orderings* of rates of behaviors and of affective evaluations of rates of behaviors.

(a) The *propositional composition* of formulas is given by the usual *propositional operators* ($\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow, \ldots$), assumed to have their usual precedence degrees.

The meaning of $\neg\langle beh\rangle$ is that it is false that the behavior $\langle beh\rangle$ has a defined rate, i.e., it is false that the behavior *beh* is in execution. The formula $\neg\langle beh\rangle[afval]$ should be read as $\neg(\langle beh\rangle[afval])$, that is, the operator of *affective evaluation* has precedence over $\neg$. The same happens with the other propositional operators, so that to express the *simultaneous affective evaluation* of two behaviors one should write $(\langle beh_i\rangle \wedge \langle beh_j\rangle)[afval]$.

(b) *Functional dependence* compound formulas express *monotonic qualitative functional dependences* between two rates of performance of behaviors. They are built with the *functional dependence operators* $\nearrow$ and $\searrow$.

The *functional dependence* compound formulas have the basic forms:

$$\langle beh_i\rangle \nearrow \langle beh_j\rangle \quad \text{and} \quad \langle beh_i\rangle \searrow \langle beh_j\rangle$$

The meaning of $\langle beh_i\rangle \nearrow \langle beh_j\rangle$ is that: (a) the rates of performance of beh_i and of beh_j are both defined, and (b) the rate of beh_j *monotonically increases* with the rate of beh_i. Correspondingly, the meaning of $\langle beh_i\rangle \searrow \langle beh_j\rangle$ is that the two rates of behaviors are related in a *monotonically decreasing* way.

The possible decorations of the *functional dependence* compound formulas with situation indexes are as follows: $\langle mag_i.beh_i\rangle \nearrow_p \langle mag_j.beh_j\rangle$ and $\langle mag_i.beh_i\rangle \searrow_p \langle mag_j.beh_j\rangle$. Notice that, since the *functional dependence* has to be determined in a single situation, the *situation indexes* of the two behaviors have to refer to the same situation.

There is no notion of *functional dependence* between affective evaluations of defined values of variables, given the assumption of *evaluation autonomy* of the material agents. So the only way *functional dependence* compound formulas may be decorated with affective evaluation operations is by indicating that the *whole functional dependence* is affectively evaluated. That is done through the general forms:

$$\big[\langle mag_i.beh_i\rangle \nearrow_p \langle mag_j.beh_j\rangle\big][mag_k.afval_k]$$

and

$$\big[\langle mag_i.beh_i\rangle \searrow_p \langle mag_j.beh_j\rangle\big][mag_k.afval_k]$$

meaning that the indicated *functional dependences* are evaluated by material agent mag_k with affective value $afval_k$.

The decoration of affectively evaluated *functional dependences* with situation indexes is also possible, e.g.:

$$\big[\langle mag_i.beh_i\rangle \nearrow_p \langle mag_j.beh_j\rangle\big][mag_k.afval_k]_q$$

(c) The *ordering* of rates of performance of behaviors, and of results of affective evaluations of rates of behaviors, is given by the partial order relation "$\preccurlyeq$".

We assume that "$\preccurlyeq$" operates uniformly on *affective values* and on *rates of performances of behaviors*. Thus, we may write both $afval_1 \preccurlyeq afval_2$ and $\langle beh_1\rangle \preccurlyeq \langle beh_2\rangle$.

For any behavior *beh* it holds that its *lowest* rate of performance (given by $\bot$), *highest* rate of performance (given by $\top$), and *medium* rate of performance (given by $\boxminus$) are ordered by the partial ordering relation "$\preccurlyeq$" as: $\bot \preccurlyeq \boxminus \preccurlyeq \top$. For the affective values resulting from the affective evaluation of rates of behavior, we extend the use of "$\preccurlyeq$" in the following way:

$$\begin{aligned}&\langle mag_1.beh_1\rangle[mag_2.afval_2]_p \preccurlyeq \langle mag_3.a_3\rangle[mag_4.afval_4]_p \Leftrightarrow \\ &\quad \langle mag_1.beh_1\rangle[mag_2.afval_2]_p \wedge \langle mag_3.a_3\rangle[mag_4.afval_4]_p \\ &\qquad \wedge\ afval_2 \preccurlyeq afval_4\end{aligned}$$

4.4 Skinner's Individual Behavior Conditioning Rules

The *fundamental relation between behaviors*, according to Skinner [5], is the relation of *operant conditioning*, that is, the relation through which an spontaneous behavior (an *operant*) is led to be performed in the context of, and in accordance to, a certain *conditioner* behavior.

The conditioning occurs because the *conditioner* behavior is assumed to represent a combination of events that is relevant for the internal functioning of the *operant agent*, so that it is capable of influencing (positively or negatively) the way that agent performs the *operant* behavior that is being conditioned.

Skinner's basic proposition (which is fully adopted by Homans) is:

> *An operant is conditioned by a conditioner whenever it happens that an increase or decrease in the rate of performance of the conditioner impacts the rate of performance of the operant behavior.*

The usual interpretation of this proposition is that the operant agent evaluates (positively or negatively) the variation (increase or decrease) in the rate of performance of the conditioner behavior, and reacts by varying accordingly the rate of performance of the operant behavior.

The *operant conditioning rules* shown in Figure 1 attempt to formally capture some of the more specific operant conditioning propositions in Skinner's behavioral psychology [5].

The "$\rightsquigarrow$" symbol denotes the impact relation between the (positive or negative) evaluation, by mag_1, of the variation (increase or decrease) in the rate of reception of the *conditioner object* obj_1 from mag_2, and the consequent variation (increase or decrease) in the rate of performance of the delivery of the object obj_1 to mag_2, by mag_1.

We may use the abbreviations shown in Figure 2 for the *positive or negative affective assessments* of the rate of the conditioner behavior, which are shown in Figure 1.

The formal expression of Skinner's rules for *emotional responses* is given by the following formulas:

- $\langle mag_2.obj_2\rangle[mag_1.+] \wedge \langle mag_2.obj_2 \bot\rangle \rightsquigarrow \langle mag_1.N\rangle$
  - meaning that the withdrawal by the material agent

$$\langle mag_1.receive(mag_1, obj_1, mag_2)\uparrow\rangle[mag_1.+] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)\uparrow\rangle$$
$$\langle mag_1.receive(mag_1, obj_1, mag_2)_\downarrow\rangle[mag_1.+] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)\uparrow\rangle$$
$$\langle mag_1.receive(mag_1, obj_1, mag_2)\uparrow\rangle[mag_1.-] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)_\downarrow\rangle$$
$$\langle mag_1.receive(mag_1, obj_1, mag_2)\downarrow\rangle[mag_1.-] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)_\downarrow\rangle$$

Figure 1. Usual interpretation of *Skinner's basic rules* formally presented.

$$\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^{+} \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle$$
$$\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^{-} \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle$$

Figure 2. Abbreviations for the positive and negative affective assessments of conditioners by the operant agent.

mag_2 of the positive reinforcer obj_2 of some operant of material agent mag_1 may release in mag_1 a negative emotional behavior N.

- $\langle mag_2.obj_2\rangle[mag_1.+]\langle mag_2.obj_2\top\rangle \rightsquigarrow \langle mag_1.P\rangle$
  - meaning that the the presentation by the material agent mag_2 of the positive reinforcer obj_2 of some operant of material agent mag_1 may release in mag_1 some degree of a positive emotional behavior P.

4.5 Homans' Conditioning Rules and their Elementary Economic Interpretation

George Homans adopted Skinner's theory as the behavioristic foundation of his *social exchange theory* [4]. In Homans' social interpretation of Skinner's conditioning rules, the *operant* is a *concrete behavior* that some agent directs toward another agent, and the *conditioner* is some *affective reward* that the latter directs toward the former. ,

In our *economic* interpretation of Homans' concept of social behavior, we recast both the *operant* and the *conditioner* behaviors to be sequences of actions of *delivering* and *receiving* objects (more precisely, *energy objects* and *chips*, respectively).

We introduce this economic interpretation as the set of *conditioning rules for individual elementary economic behaviors* shown in Figure 3. The rules are characterized by the type of the conditioning acting on the operant behavior ("+" or "−") and by the level of activity of the conditioner ("⊤", "⊟" or "⊥"). The informal reading of those rules are as follows:

1. Rule $BC_{(+,\boxminus)}$: If mag_1 evaluates *positively* (+) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2, and mag_2 delivers obj_2 to mag_1 at *a regular temporal rate* (⊟) then: the more frequently mag_2 delivers obj_2 to mag_1, the more frequently will mag_1 deliver obj_1 to mag_2. Formally:

2. Rule $BC_{(+,\top)}$: If mag_1 evaluates *positively* (+) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2, and mag_2 delivers obj_2 to mag_1 at a *very high temporal rate* (⊤) then: the more frequently mag_2 delivers obj_2 to mag_1, the less frequently will mag_1 deliver obj_1 to mag_2. Formally:

3. Rule $BC_{(+,\perp)}$: If mag_1 evaluates *positively* (+) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2, and mag_2 delivers obj_2 to mag_1 at a *very low temporal rate* (⊥) then: the more frequently mag_2 delivers obj_2 to mag_1, the more frequently will mag_1 deliver obj_1 to mag_2. Formally:

4. Rule BC_{-}: If mag_1 evaluates *negatively* (−) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2 then: the more frequently mag_2 delivers obj_2 to mag_1, the less frequently will mag_1 deliver obj_1 to mag_2. Formally:

5. Rule $BC_{\downarrow}$: Any *increase* in frequency of a particular behavior beh_1 by mag entails by that very fact a *decrease* in the frequency of any alternative behavior beh_2 that mag can perform. In the rule, $\text{beh}[mag]$ denotes the set of behaviors that mag is capable of performing.

5. Individual Elementary Economic Exchanges

We call *individual elementary economic exchanges* the elementary economic exchanges performed between individual material agents. They should be distinguished from the *group* elementary economic exchanges, introduced in Section 7.

In this section, besides the concept of *individual elementary economic exchanges*, and its formal notation, we formally introduce the concepts of *material outcome function*, *material equilibrium* and *operational equilibrium* of individual elementary economic exchanges.

$$\frac{\begin{array}{c}\langle mag_1.receive(mag_1,obj_2,mag_2)\rangle \twoheadrightarrow^{+} \langle mag_1.deliver(mag_1,obj_1,mag_2)\rangle \\ \langle mag_2.deliver(mag_2,obj_2,mag_1)\rangle\boxminus\end{array}}{\langle mag_2.deliver(mag_2,obj_2,mag_1)\rangle \nearrow \langle mag_1.deliver(mag_1,obj_1,mag_2)\rangle}\; BC_{(+,\boxminus)}$$

$$\frac{\begin{array}{c}\langle mag_1.receive(mag_1,obj_2,mag_2)\rangle \twoheadrightarrow^{+} \langle mag_1.deliver(mag_1,obj_1,mag_2)\rangle \\ \langle mag_2.deliver(mag_2,obj_2,mag_1)\rangle\top\end{array}}{\langle mag_2.deliver(mag_2,obj,mag_1)\rangle \searrow \langle mag_1.deliver(mag_1,beh,mag_2)\rangle}\; BC_{(+,\top)}$$

$$\frac{\begin{array}{c}\langle mag_1.receive(mag_1,obj_2,mag_2)\rangle \twoheadrightarrow^{+} \langle mag_1.deliver(mag_1,obj_1,mag_2)\rangle \\ \langle mag_2.deliver(mag_2,obj,mag_1)\rangle\bot\end{array}}{\langle mag_2.deliver(mag_2,obj,mag_1)\rangle \nearrow \langle mag_1.deliver(mag_1,beh,mag_2)\rangle}\; BC_{(+,\bot)}$$

$$\frac{\langle mag_1.receive(mag_1,obj_2,mag_2)\rangle \twoheadrightarrow^{-} \langle mag_1.deliver(mag_1,obj_1,mag_2)\rangle}{\langle mag_2.deliver(mag_2,obj,mag_1)\rangle \searrow \langle mag_1.deliver(mag_1,beh,mag_2)\rangle}\; BC_{-}$$

$$\frac{beh_i, beh_j \in \mathrm{beh}[mag]}{\langle mag.beh_i\rangle \searrow_{i\neq j} \langle mag.beh_j\rangle}\; BC_{\downarrow}$$

Figure 3. Operant conditioning rules for individual elementary economic behaviors.

5.1 Terms for Denoting Individual Elementary Economic Exchanges

An *individual elementary economic exchange* is a *pair of operant conditionings* acting between two material agents, mag_1 and mag_2, so that the delivery of the object obj_1, by mag_1 to mag_2 acts as a *conditioner* to the delivery of the object obj_2 by mag_2 to mag_1, and vice-versa.

We say that an *individual elementary economic exchange* between mag_1 and mag_2, involving the exchange of the objects obj_1 and obj_2 between them, is performed in the *doubly positive mode of exchange* if and only if:

1. obj_1 is an *energy object* and obj_2 is a *chip*, or vice-versa;
2. $\langle mag_1.receive(mag_1,obj_2,mag_2)\rangle \twoheadrightarrow^{+} \langle mag_1.deliver(mag_1,obj_1,mag_2)\rangle$
3. $\langle mag_2.receive(mag_2,obj_1,mag_1)\rangle \twoheadrightarrow^{+} \langle mag_2.deliver(mag_2,obj_2,mag_1)\rangle$

We denote such a *doubly positively reinforced* individual elementary economic exchange by:

$$\langle mag_1/obj_1\rangle \;{}_{+}\!\rightleftarrows^{+}\; \langle mag_2/obj_2\rangle$$

The vicissitudes of the performance of an *individual elementary economic exchange* (e.g., variations in the rate of reception of conditioners) may lead the material agents involved in it to change the way they evaluate the conditioners they receive from their partners. As a result, the following are other *modes of exchange* that may occur during the performance of an individual elementary economic exchange:

- *mixed* modes of exchange:

$$\langle mag_1/obj_1\rangle \;{}_{-}\!\rightleftarrows^{+}\; \langle mag_2/obj_2\rangle$$

and

$$\langle mag_1/obj_1\rangle \;{}_{+}\!\rightleftarrows^{-}\; \langle mag_2/obj_2\rangle$$

- *doubly negative* mode of exchange:

$$\langle mag_1/obj_1\rangle \;{}_{-}\!\rightleftarrows^{-}\; \langle mag_2/obj_2\rangle$$

5.2 The Material Outcome of Individual Elementary Economic Exchanges

Let mag_1 and mag_2 be two material agents performing a doubly positive individual elementary economic exchange of the form $ie2exch = \langle mag_1/obj_1\rangle \;{}_{+}\!\rightleftarrows^{+}\; \langle mag_2/obj_2\rangle$. Let $\mathrm{obj}^t[mag_1] \in \mathbb{N}\times\mathbb{N}$ and $\mathrm{obj}^t[mag_2] \in \mathbb{N}\times\mathbb{N}$ be the amounts of objects of types Obj_1 and Obj_2 that each such material agent respectively has at the time t. The *material outcome* of the individual elementary economic exchange *ie2exch*, obtained between the times t and $t' > t$ is given by:

$$\text{moutcome}[ie2exch]_t^{t'} = (\text{obj}^{t'}[mag_1] - \text{obj}^t[mag_1], \text{obj}^{t'}[mag_2] - \text{obj}^t[mag_2])$$

where, for any material agent mag and any times t and $t' > t$, we have that[3]:

$$\text{obj}^{t'}[mag] - \text{obj}^t[mag] = (\text{obj}^{t'}[mag][1] - \text{obj}^t[mag][1], \text{obj}^{t'}[mag][2] - \text{obj}^t[mag][2])$$

with $\text{moutcome}[ie2exch]_t^{t'} \in \mathbb{Z} \times \mathbb{Z}$, so that the material outcome of an individual elementary economic exchange can have negative components.

5.3 Operational Equilibrium of Individual Elementary Economic Exchanges

We say that the *individual elementary economic exchange*:

$$ie2exch = \langle mag_1/obj_1 \rangle \ {}_{+}\!\rightleftarrows^{+} \langle mag_2/obj_2 \rangle$$

which is performed at a time t, is *operationally equilibrated* at that time if and only if it holds, for each of the behaviors $\langle mag_1/obj_1 \rangle$ and $\langle mag_2/obj_2 \rangle$, that they are performed at a *medium rate* at that time, that is: $\langle mag_1/obj_1 \rangle^t \boxminus$ and $\langle mag_2/obj_2 \rangle^t \boxminus$.

We denote by $\text{opequil}^\tau[ie2exch]$ the fact that the individual elementary economic exchange *ie2exch* is operationally equilibrated at the time τ.

6. Individual Elementary Economic Processes

An *individual elementary economic process* is a time-indexed sequence of individual elementary economic exchanges, performed by two given material agents, as explained in this section.

6.1 Sequential Composition of Operant Conditionings of Individual Elementary Economic Behaviors

Operant conditionings between individual elementary economic behaviors can be *sequentially composed*, in the sense that a behavior beh_2 that is conditioned by a behavior beh_1 can, itself, condition a behavior beh_3. In such situation, one can say that behavior beh_1 also conditions behavior beh_3.

Figure 4 shows the general rule defining such *sequential composition*, considering the possible positive or negative conditionings that the behaviors may have on each other, that is: $c_1, c_2 \in \{+, -\}$.

6.2 Sequential Composition of Individual Elementary Economic Exchanges

Individual elementary economic exchanges can also be sequentially composed. Figure 5 shows the general rule defining such sequential composition, considering the possible positive or negative conditionings that the behaviors may have in each exchange, that is: $c_{1,2}, c_{2,1}, c_{2,3}, c_{3,2} \in \{+, -\}$.

Notice that the material agent $\langle mag \rangle_2$ can act as an *intermediary* between the material agents mag_1 and mag_3 only if it can handle separately the exchange of *two* objects, obj_2 and obj'_2, each corresponding to each of its partner (even if those objects are of the same type).

6.3 Chains of Individual Elementary Economic Exchanges

We call *chain of individual elementary economic exchanges* any finite sequence of compositions of individual elementary economic exchanges, of the form:

$$\begin{aligned} ie3chain = & \left[\langle mag_1/obj_1 \rangle \ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle mag_2/obj_2 \rangle\right] \otimes \\ & \left[\langle mag_2/obj'_2 \rangle \ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle mag_3/obj_3 \rangle\right] \otimes \ldots \otimes \\ & \left[\langle mag_{n-1}/obj'_{n-1} \rangle \ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle mag_n/obj_n \rangle\right] \end{aligned}$$

where $\text{len}[ie3chain] = n - 1$, for $n \geq 2$, is the *length* of the chain.

6.4 Individual Elementary Economic Processes, Formally Defined

We call *individual elementary economic process* any finite time-indexed sequence of *chains of individual elementary economic exchanges*, where all chains in the sequence are of the same form and are performed by the same set of material agents (say $\{mag_1, mag_2, \ldots, mag_n\}$).

We denote *individual elementary economic processes* by:

$$ie2proc^t = ie3chain^0; ie3chain^1; \ldots; ie3chain^t$$

where $t \geq 0$ and for each $\tau \leq t$ it holds that:

$$\begin{aligned} ie3chain^\tau = & \left[\langle mag_1/obj_1 \rangle^\tau \ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle mag_2/obj_2 \rangle^\tau\right] \otimes \\ & \left[\langle mag_2/obj'_2 \rangle^\tau \ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle mag_3/obj_3 \rangle^\tau\right] \otimes \ldots \otimes \\ & \left[varmag_{n-1}/obj'^{\ \tau}_{n-1} \ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle mag_n/obj_n \rangle^\tau\right] \end{aligned}$$

where:

- each $\langle mag_i/obj_i \rangle^\tau \ {}_{c_{j,i}}\!\rightleftarrows^{c_{i,j}} \langle mag_j/obj_j \rangle^\tau$ is an *individual elementary economic exchange* that may occur at the time τ;
- there is *no cycle* in $ie2iproc^t$ (that is, $mag_i \neq mag_j$, for every $i, j \leq n$);
- t is the *length* of *ie2proc*, and n, the length of the chains, is the *width* of *ie2proc*.

We say that, for $0 \leq \tau \leq t$, the chain $ie3chain^\tau$ is the *process step* that occurs at the time τ in $ie2proc^t$.

6.5 Material Outcome of Individual Elementary Economic Processes

Given the individual elementary economic process $ie2iproc^t$ we define its *material outcome*, regarding the time interval

[3] In the following, any reference to the i-th component of a tuple X is denoted by $X[i]$.

$$\frac{\langle beh_1\rangle \twoheadrightarrow^{c_1} \langle beh_2\rangle \qquad \langle beh_2\rangle \twoheadrightarrow^{c_2} \langle beh_3\rangle}{[\langle beh_1\rangle \twoheadrightarrow^{c_1} \langle beh_2\rangle] \otimes [\langle beh_2\rangle \twoheadrightarrow^{c_2} \langle beh_3\rangle]}\ SCB$$

Figure 4. General rule for the sequential composition of operant conditionings of individual elementary economic behaviors.

$$\frac{\langle mag_1/obj_1\rangle\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle mag_2/obj_2\rangle \qquad \langle mag_2/obj_2\rangle\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle mag_3/obj_3\rangle}{\left[\langle mag_1/obj_1\rangle\ {}_{c_{21}}\!\rightleftarrows^{c_{12}} \langle mag_2/obj_2\rangle\right] \otimes \left[\langle mag_2/obj_2'\rangle\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle mag_3/obj_3\rangle\right]}\ SCE$$

Figure 5. General rule for sequential composition of individual elementary economic exchanges.

$[\tau,\tau']$, for $0 \le \tau < \tau' \le t$, in the following way:

$$\begin{aligned}&\mathrm{moutcome}[\mathit{ie2proc}^t]_\tau^{\tau'} = \\ &\quad(\mathrm{obj}^{\tau'}[mag_1] - \mathrm{obj}^{\tau}[mag_1], \ldots, \mathrm{obj}^{\tau'}[mag_n] - \mathrm{obj}^{\tau}[mag_n])\end{aligned}$$

We also define a *reduced form* of the material outcome of the individual elementary economic process *ie2proc*, as:

$$\begin{aligned}&\mathrm{redmoutcome}[\mathit{ie2proc}]_\tau^{\tau'} = \\ &\quad(\mathrm{obj}^{\tau'}[mag_1] - \mathrm{obj}^{\tau}[mag_1], \mathrm{obj}^{\tau'}[mag_n] - \mathrm{obj}^{\tau}[mag_n])\end{aligned}$$

6.6 Material Equilibrium of an Individual Elementary Economic Process

We say that an individual elementary economic process $\mathit{ie2proc}^t$ is in *material equilibrium*, regarding a time interval $[\tau,\tau']$, if and only if its *material outcome* for that time interval is *null*, that is, $\mathrm{moutcome}[\mathit{ie2proc}^t]_\tau^{\tau'} = (0,0,\ldots,0)$.

Material equilibrium means, thus, that the *net amount of objects* that were exchanged and consumed by the material agents that participate in $\mathit{ie2proc}^t$, during that interval, is null.

6.7 Operational Equilibrium of a Chain of Individual Elementary Economic Processes

The *operational equilibrium* of a chain of individual elementary economic process, at a given time, is given by the fact that each of the exchanges that compose it is operationally equilibrated at that time. That is, the chain of individual elementary economic processes:

$$\begin{aligned}\mathit{ie3chain} = {} & \left[\langle mag_1/obj_1\rangle\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle mag_2/obj_2\rangle\right] \otimes \\ & \left[\langle mag_2/obj_2'\rangle\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle mag_3/obj_3\rangle\right] \otimes \ldots \otimes \\ & \left[varmag_{n-1}/obj_{n-1}'\ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle mag_n/obj_n\rangle\right]\end{aligned}$$

is operationally equilibrated at the time τ if and only if each of the individual elementary economic exchanges that constitute it is operationally equilibrated at that time:

$$\forall \langle mag_i/obj_i\rangle \in \mathit{ie3chain}\,(\mathrm{opequil}^\tau[\langle mag_i/obj_i\rangle])$$

We denote by $\mathrm{opequil}^\tau[\mathit{ie3chain}]$ the fact that *ie3chain* is operationally equilibrated at the time τ.

6.8 Operational Equilibrium of Individual Elementary Economic Processes

We say that an individual elementary economic process is *operationally equilibrated*, at a given time, if and only if the *process step* that occurs in that individual elementary economic process, at that time, is operationally equilibrated. That is, the individual elementary economic process:

$$\mathit{ie2proc}^t = \mathit{ie3chain}^0; \mathit{ie3chain}^1; \ldots; \mathit{ie3chain}^t$$

is operationally equilibrated at the time τ (with $0 \le \tau \le t$) if and only if the process step:

$$\begin{aligned}\mathit{ie3chain}^\tau = {} & \left[\langle mag_1/obj_1\rangle^\tau\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle mag_2/obj_2\rangle^\tau\right] \otimes \\ & \left[\langle mag_2/obj_2'\rangle^\tau\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle mag_3/obj_3\rangle^\tau\right] \otimes \ldots \otimes \\ & \left[varmag_{n-1}/obj_{n-1}'{}^\tau\ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle mag_n/obj_n\rangle^\tau\right]\end{aligned}$$

is operationally equilibrated, that is, $\mathrm{opequil}^\tau[\mathit{ie3chain}]$.

We denote by $\mathrm{opequil}^\tau[\mathit{ie2proc}]$ the fact that $\mathit{ie2proc}^t$ is operationally equilibrated at the time $\tau \le t$. We denote by $\mathrm{opequil}_\tau^{\tau'}[\mathit{ie2proc}]$ the fact that $\mathit{ie2proc}^t$ is operationally equilibrated at each of the time instants of the interval $[\tau,\tau']$.

6.9 Relationship between the Operational and the Material Equilibrium of an Individual Elementary Economic Process

We remark that there is no necessary relation between the *operational equilibrium* and the *material equilibrium* of an individual elementary economic process.

The reason is that *material equilibrium* of individual elementary economic processes is defined in terms of variations in the *amounts of objects* that the involved material agents have at the beginning and at the end of the considered process, while the *operational equilibrium* of individual elementary economic processes is defined in terms of the *rates of performances* of the exchange behaviors that constitute them.

7. Group Elementary Economic Behaviors and Exchanges

In this section, we extend for *social groups* of material agents the concepts of behavior and exchange that were introduced above for *individual* material agents.

7.1 Group Elementary Economic Behaviors and Exchanges, Formally Defined

The general form of individual elementary economic exchange introduced in Sect. 6, namely,

$$\langle mag_1/obj_1\rangle\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}}\ \langle mag_2/obj_2\rangle$$

is constituted by a combination of elementary economic behaviors performed by two *individual* material agents (mag_1 and mag_2).

We can naturally extend the concept of elementary economic behavior to *groups* of material agents, considering that each such group operates as a *unity*, collectively delivering objects to another group, and collectively being reinforced by the reception of objects from that other group.

Let Mag and Mag' be two *groups of material agents*. The operation $deliver^t(Mag,Obj,Mag')$ indicates, then, the delivery of the set of objects Obj by Mag to Mag', at the time t, and the operation $receive^t(Mag,Obj,Mag')$ indicates that Mag receives the set Obj from Mag' at the time t.

A *group elementary economic exchange* between Mag_1 and Mag_2, exchanging sets of objects Obj_1 and Obj_2, is denoted by: $\langle Mag_1/Obj_1\rangle\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}}\ \langle Mag_2/Obj_2\rangle$, with reinforcement signs $c_{1,2},c_{2,1}\in\{+,-\}$.

7.2 Material Outcome of Group Elementary Economic Processes

Given the group elementary economic process $ge2iproc^t$ we define its *material outcome*, regarding the time interval $[\tau,\tau']$, for $0\le\tau<\tau'\le t$, in the following way:

$$\mathrm{moutcome}[ge2proc^t]_{\tau}^{\tau'} = (\mathrm{Obj}^{\tau'}[Mag_1]-\mathrm{Obj}^{\tau}[Mag_1],\dots,\mathrm{Obj}^{\tau'}[Mag_n]-\mathrm{Obj}^{\tau}[Mag_n])$$

We also define a *reduced form* of the material outcome of the individual elementary economic process $ge2proc$, as:

$$\mathrm{redmoutcome}[ge2proc]_{\tau}^{\tau'} = (\mathrm{Obj}^{\tau'}[Mag_1]-\mathrm{Obj}^{\tau}[Mag_1],\mathrm{Obj}^{\tau'}[Mag_n]-\mathrm{Obj}^{\tau}[Mag_n])$$

7.3 Material Equilibrium of a Group Elementary Economic Process

We say that a group elementary economic process $ge2proc^t$ is in *material equilibrium*, regarding a time interval $[\tau,\tau']$ if and only if its *material outcome function* for that time interval is *null*, that is, $\mathrm{moutcome}[ge2proc^t]_{\tau}^{\tau'}=(0,0,\dots,0)$.

Material equilibrium means, thus, that the *net amount of objects* that were exchanged and consumed by the material groups of material agents that participate in $ge2proc^t$, during that interval, is null.

7.4 Operational Equilibrium of Group Elementary Economic Exchanges

We say that the group elementary economic exchange:

$$ge2exch = \langle Mag_1/Obj_1\rangle\ {}_{+}\!\rightleftarrows^{+}\ \langle Mag_2/Obj_2\rangle$$

is *operationally equilibrated* at a time t if and only if it holds, for each of the behaviors of Mag_1 and Mag_2, that they are performed at a *medium rate* at that time, that is, $\langle Mag_1/obj_1\rangle^t\boxminus$ and $\langle Mag_2/obj_2\rangle^t\boxminus$.

We denote by $\mathrm{opequil}^{\tau}[ge2exch]$ that the group elementary economic exchange $ge2exch^t$ is operationally equilibrated at the time $\tau\le t$.

Notice that the condition of *operational equilibrium* of a *group* elementary economic exchange is defined on the basis of a notion of *group* rate of performance of group behaviors (which we leave undefined here). Thus, the condition of operational equilibrium of a *group* elementary economic exchange does *not* require that the *individual* elementary economic exchanges that compose it be operationally equilibrated.

8. Group Elementary Economic Processes

In this section, we extend to *social groups* of material agents the concept of *elementary economic process* introduced above for *individual* material agents.

8.1 Chains of Group Elementary Economic Exchanges

We call *chain of group elementary economic exchanges* any finite sequence of compositions of group elementary economic exchanges, of the form:

$$\begin{aligned} ge3chain = {} & \left[\langle Mag_1/Obj_1\rangle\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}}\ \langle Mag_2/Obj_2\rangle\right]\otimes \\ & \left[\langle Mag_2/Obj_2'\rangle\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}}\ \langle Mag_3/Obj_3\rangle\right]\otimes\dots\otimes \\ & \left[varMag_{n-1}/Obj_{n-1}'\ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}}\ \langle Mag_n/Obj_n\rangle\right] \end{aligned}$$

where $\mathrm{len}[ge3chain]=n-1$, for $n\ge 2$, is the *length* of the chain.

8.2 Group Elementary Economic Processes, Formally Defined

We call *group elementary economic process* any finite time-indexed sequence of *chains of group elementary economic exchanges*, where all chains in the sequence are of the same form, are performed by the same set of groups of material agents (say $\{Mag_1,Mag_2,\dots,Mag_n\}$).

We denote *group elementary economic processes* by:

$$ge2proc^t = ge3chain^0;ge3chain^1;\dots;ge3chain^t$$

where $t\ge 0$ and for each $\tau\le t$ it holds that:

$$\begin{aligned} ge3chain^{\tau} = {} & \left[\langle Mag_1/Obj_1\rangle^{\tau}\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}}\ \langle Mag_2/Obj_2\rangle^{\tau}\right]\otimes \\ & \left[\langle Mag_2/Obj_2'\rangle^{\tau}\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}}\ \langle Mag_3/Obj_3\rangle^{\tau}\right]\otimes\dots\otimes \\ & \left[varMag_{n-1}/Obj_{n-1}'{}^{\tau}\ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}}\ \langle Mag_n/Obj_n\rangle^{\tau}\right] \end{aligned}$$

where:

- each $\langle Mag_i/Obj_i\rangle^{\tau}\ {}_{c_{j,i}}\!\rightleftarrows^{c_{i,j}}\ \langle Mag_j/Obj_j\rangle^{\tau}$ is a *group elementary economic exchange* that may occur at the time τ;

- there is *no cycle* in $ge2iproc^t$ (that is, $Mag_i \neq Mag_j$, for every $i, j \leq n$);
- t is the *length* of *ge2proc*, and n, the length of the chains, is the *width* of *ge2proc*.

We say that, for $0 \leq \tau \leq t$, the chain $ge3chain^\tau$ is the *process step* that occurs at the time τ in $ge2proc^t$.

8.3 Material Outcome of Group Elementary Economic Processes

The *material outcome* of a group elementary economic process, relatively to the time interval $[\tau, \tau']$, with $0 \leq \tau < \tau' \leq t$, is defined in the following way:

$$\mathrm{moutcome}[ge2proc^t]_\tau^{\tau'} = (\mathrm{Obj}^{\tau'}[Mag_1] - \mathrm{Obj}^\tau[Mag_1], \ldots, \mathrm{Obj}^{\tau'}[Mag_n] - \mathrm{Obj}^\tau[Mag_n])$$

with the extension of the operation of subtraction to *sets of objects* held by groups of material agents, under the assumption that each such set of material agents is capable of accounting for the number of sets of objects that it has received from, and sent to, other sets of material agents.

The reduced form of the *material outcome* of the group elementary economic process *ge2proc* is the given by:

$$\mathrm{redmoutcome}[ge2proc]_\tau^{\tau'} = (\mathrm{Obj}^{\tau'}[Mag_1] - \mathrm{Obj}^\tau[Mag_1], \mathrm{Obj}^{\tau'}[Mag_n] - \mathrm{Obj}^\tau[Mag_n])$$

8.4 Material Equilibrium of a Group Elementary Economic Process

We say that a group elementary economic process $ge2proc^t$ is in *material equilibrium*, regarding a time interval $[\tau, \tau']$ if and only if its *material outcome* for that time interval is *null*, that is, $\mathrm{moutcome}[ge2proc^t]_\tau^{\tau'} = (0, 0, \ldots, 0)$.

8.5 Operational Equilibrium of a Chain of Group Elementary Economic Processes

The *operational equilibrium* of a chain of group elementary economic process, at a given time, is given by the fact that each of the exchanges that compose it is operationally equilibrated at that time. That is, the chain of group elementary economic processes:

$$\begin{aligned} ge3chain = &\left[\langle Mag_1/Obj_1\rangle \ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle Mag_2/Obj_2\rangle\right] \otimes \\ &\left[\langle Mag_2/Obj_2'\rangle \ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle Mag_3/Obj_3\rangle\right] \otimes \ldots \otimes \\ &\left[varMag_{n-1}/Obj_{n-1}' \ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle Mag_n/Obj_n\rangle\right] \end{aligned}$$

is *operationally equilibrated* at the time τ if and only if each of the group elementary economic exchanges that constitute it is operationally equilibrated at that time.

That is, taking:

$$ge3exch_{i,j} = \langle Mag_i/Obj_i\rangle \ {}_{c_{j,i}}\!\rightleftarrows^{c_{i,j}} \langle Mag_j/Obj_j\rangle$$

it holds that:

$$\forall ge3exch_{i,j} \in ge3chain \left(\mathrm{opequil}^\tau[ge3exch_{i,j}]\right)$$

We denote by $\mathrm{opequil}^\tau[ge3chain]$ the fact that *ge3chain* is operationally equilibrated at the time τ.

8.6 Operational Equilibrium of Group Elementary Economic Processes

We say that a group elementary economic process is *operationally equilibrated*, at a given time, if and only if the *process step* that occurs in that group elementary economic process, at that time, is operationally equilibrated. That is, the group elementary economic process:

$$ge2proc^t = ge3chain^0; ge3chain^1; \ldots; ge3chain^t$$

is operationally equilibrated at the time τ (with $0 \leq \tau \leq t$) if and only if the process step:

$$\begin{aligned} ge3chain^\tau = &\left[\langle Mag_1/Obj_1\rangle^\tau \ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle Mag_2/Obj_2\rangle^\tau\right] \otimes \\ &\left[\langle Mag_2/Obj_2'\rangle^\tau \ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \langle Mag_3/Obj_3\rangle^\tau\right] \otimes \ldots \otimes \\ &\left[varMag_{n-1}/Obj_{n-1}'{}^\tau \ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle Mag_n/Obj_n\rangle^\tau\right] \end{aligned}$$

is operationally equilibrated, that is, $\mathrm{opequil}^\tau[ge3chain]$.

We denote by $\mathrm{opequil}^\tau[ge2proc]$ the fact that $ge2proc^t$ is operationally equilibrated at the time τ. We denote by $\mathrm{opequil}_\tau^{\tau'}[ge2proc]$ the fact that $ge2proc^t$ is operationally equilibrated at each time instant of the interval $[\tau, \tau']$.

9. Elementary Economic Systems

Informally, the *elementary economic system* of a material agent society is, at a certain time, the set of group elementary economic processes that occur, at that time, among groups of material agents of that material agent society, possibly with some groups of agents participating in more than one group elementary economic process.

9.1 Elementary Economic Systems, Formally Defined

The *elementary economic system* of a material agent society *MAgSoc* whose population of material agents is *Pop*, is a time-indexed structure:

$$EES^t_{MAgSoc} = (Group^t, Obj^t, GE2Beh^t, GE2Exch^t, GE2Proc^t)$$

where[4]:

- $Group^t \subseteq \wp(Pop)$ is the family of *groups of material agents* of *Pop* that can participate in the elementary economic group processes of $GE2Proc^t$;
- $Obj^t \subseteq \wp(\mathbf{Obj})$ is the family of *sets of objects* that the groups of material agents of $Group^t$ can exchange between them during the performance of the group elementary economic processes of $GE2Proc^t$;
- $GE2Beh^t$ is the set of *group elementary economic behaviors* that the groups of material agents of $Group^t$

[4] $\wp(X)$ denotes the powerset of the set X.

can perform during the performance of the group elementary economic processes of $GE2Proc^t$;

- $E2GExch^t$ is the set of *group elementary economic exchanges* that the groups of material agents of $Group^t$ can perform during the performance of the group elementary economic processes of $GE2Proc^t$;
- $GE2Proc^t$ is the set of *group elementary economic processes* that the groups of material agents of $Group^t$ can perform in *MAgSoc* at the time t.

9.2 Material Outcome of Elementary Economic Systems

Given the elementary economic system:

$$EES^t_{MAgSoc} = (Group^t, Obj^t, E2GBeh^t, E2GExch^t, E2GProc^t)$$

of the material agent society *MAgSoc*, we define the *material outcome* of EES^t_{MAgSoc} as follows: whenever the set $EEGProc^t$ of elementary economic processes is given by $EEGProc^t = \{ge2proc_1, \ldots, ge2proc_n\}$, with $n > 1$, then the *material outcome* of EES^t_{MAgSoc} is given by the sum total of the *material outcomes* of its group elementary economic processes, that is:

$$\text{moutcome}[EES^t_{MAgSoc}] = \textstyle\sum_{i=1}^{i=n} \text{moutcome}[ge2proc_i]$$

where the summation is performed in a component-wise way.

9.3 Operational Equilibrium of Elementary Economic Systems

We say that the elementary economic system:

$$EES^t_{MAgSoc} = (Group^t, Obj^t, GE2Beh^t, GE2Exch^t, GE2Proc^t)$$

is *operationally equilibrated* at the time τ (with $0 \leq \tau \leq t$) if and only if each of the group elementary economic process that constitute it is operationally equilibrated at that time, that is:

$$\forall ge2proc \in GE2Proc^\tau (\text{equil}^\tau[ge2proc])$$

We denote by $\text{equil}^\tau[EES_{MAgSoc}]$ the fact that the elementary economic system EES^t_{MAgSoc} is equilibrated at time τ. We denote by $\text{equil}_\tau^{\tau'}[EES_{MAgSoc}]$ the fact that EES_{MAgSoc} is equilibrated at each time instant of the interval $[\tau, \tau']$.

10. Ecosystems: A Case Study in Elementary Economical Analysis

We provide here elements for the *elementary economical analysis of ecosystems*. More precisely, we consider: (a) *ecosystems* as material agent societies; and (b) the *interactional structure* that constitute the operational part of the organizational structure of an ecosystem as an *elementary economical system*.

The casting of ecosystems as agent societies was introduced in [6]. The proposal for considering the *interactional structure* of an ecosystem as an *elementary economical system* is presented here for the first time.

Figure 6 pictures a general model of ecosystems. We base on it the *elementary economical analysis* that follows.

As in [6], we state that, formally, an *ecosystem* is a material agent society $EcoSys^t = (Pop^t, Org^t, MEnv^t)$ where, for each time t:

- Pop^t is the *populational structure* of the ecosystem;
- Org^t is the *organizational structure* of the ecosystem;
- $MEnv^t$ is the ecosystem's *material environment*.

So, regarding the general model of ecosystem illustrated in Figure 6, the detailment of its material agent society-based model can be given as in Figure 7.

Clearly:

- *Detrivores*, *Carnivores*, *Herbivores* and *Plants* constitute the *populational groups* of the ecosystems;
- the *Sun* and the *Soil* are the source of all *energy* consumed by the populational groups of ecosystems;
- the *Atmosphere* and the *Soil* operate as *transportation means* for the O_2, CO_2, *Water* and *Nutrients* that the populational groups exchange between them;
- individuals of a populational group eating individuals of another populational group is also a means for the former group to receive energy from the latter one;
- the set of exchange processes operating in the ecosystem constitute a complex network;
- there are more than two types of objects being exchanged in the system.

Also, from the economical point of view, such system surpasses the conditions stipulated by the concepts introduced in the present paper, and an *elementary economical analysis* of ecosystems can only partially account for it. So, we restrict ourselves here to modeling just the following *elementary economical exchanges* present in Figure 6:

1) $ge2exch_1 =$
$$\langle Detrivores/(Water + Nutrients) : Soil\rangle \;{}_{+}\!\rightleftarrows^{+} \langle Plants/Plants\rangle$$

where:

- $\langle Detrivores/(Water + Nutrients) : Soil\rangle$ denotes that *Detrivores* deliver objects of types *Water* and *Nutrients* through the *Soil* to *Plants*;
- $\langle Plants/Plants\rangle$ denotes that *Plants* deliver objects of type *Plants* directly to (be eaten by) *Detrivores*.

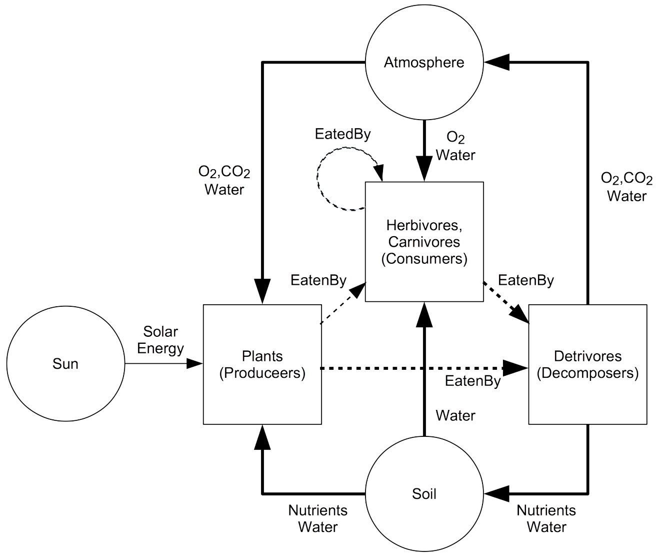

Figure 6. The general model of ecosystems, based on [7].

- $Pop^t = (Plants^t, Herbivores^t, Carnivores^t, Detrivores^t)$
  - a tuple of sub-populations;
- $Org^t = (O_2^t, CO_2^t, Water^t, Nutrients^t, EatenBy^t, SolarEnergy^t)$
  - a tuple of sets of exchange links, through which are exchanged the corresponding substances;
- $MEnv^t = (Atmosphere^t, Soil^t, Sun^t)$
  - a tuple of material objects;

with:

- $O_2^t = \{(Detrivores^t, Atmosphere^t), (Atmosphere^t, Plants^t), (Atmosphere^t, Herbivores^t), (Atmosphere^t, Carnivores^t)\}$
- $CO_2 = \{(Detrivores^t, Atmosphere^t), (Atmosphere^t, Plants^t)\}$
- $Water^t = \{(Detrivores^t, Atmosphere^t), (Atmosphere^t, Plants^t), (Atmosphere^t, Herbivores^t), (Atmosphere^t, Carnivores^t), (Detrivores^t, Soil^t), (Soil^t, Plants^t), (Soil^t, Herbivores^t), (Soil^t, Carnivores^t)\}$
- $Nutrients^t = \{(Detrivores^t, Soil^t), (Soil^t, Plants^t)\}$
- $EatenBy^t = \{(Herbivores^t, Detrivores^t), (Carnivores^t, Detrivores^t), (Plants^t, Detrivores^t), (Plants^t, Herbivores^t), (Plants^t, Carnivores^t), (Carnivores^t, Carnivores^t)\}$
- $SolarEnergy^t = \{(Sun^t, Plant^t)\}$

Figure 7. The detailment of the material agent society-based modeling of the ecosystem shown in Figure 6.

2) $ge2exch_2$ =
$\langle Detrivores/Water : Soil\rangle\ _{+}\rightleftarrows^{+}$
$\langle (Herbivores + Carnivores)/(Herbivores + Carnivores)\rangle$

3) $ge2exch_3$ =
$\langle Detrivores/(O_2 + Water) : Atmosphere\rangle\ _{+}\rightleftarrows^{+}$
$\langle (Herbivores + Carnivores)/(Herbivores + Carnivores)\rangle$

4) $ge2exch_4$ =
$\langle Detrivores/(O_2 + CO_2 + Water) : Atmosphere\rangle\ _{+}\rightleftarrows^{+}$
$\langle Plants/Plants\rangle$

On the other hand, the sole *elementary economic process* present in the general model of ecosystems shown in Figure 6 is the one given by:

$e2proc$ =
$[\langle Plants/Plants\rangle\ _{+}\rightleftarrows^{+}\ \langle Detrivores/(Water + Nutrients) : Soil\rangle]$
$\otimes$
$[\langle Detrivores/(O_2 + Water) : Atmosphere\rangle\ _{+}\rightleftarrows^{+}$
$\langle (Herbivores + Carnivores)/(Herbivores + Carnivores)\rangle]$

Thus, even with the *limited modeling capability* provided by the *elementary concepts* of economic systems introduced in this paper, we can cast at least a *part* of the general model of ecosystems given in Figure 6 in the form of an *elementary economic system*, as follows:

$$EES^t_{EcoSys} = (Group^t, Obj^t, GE2Beh^t, GE2Exch^t, GE2Proc^t)$$

with:
• $Group^t = Pop^t$
- because all of the populational sub-groups of the ecosystem participate, as *elementary economical groups*, in the elementary economic system;
• Obj^t =
$\{O_2, CO_2, Water, Nutrients, Plants, Herbivores, Carnivores\}$
- which are the *objects* exchanged by the elementary economical groups;
• $GE2Beh^t = DeliverBehaviors \cup ReceiveBehaviors$
where:
$DeliverBehaviors$ =
$\{deliver(Detrivores, Water, Carnivores)$,
$deliver(Detrivores, O_2, Carnivores)$,
$deliver(Detrivores, CO_2, Carnivores)$,
$deliver(Detrivores, Water, Herbivores)$,
$deliver(Detrivores, O_2, Herbivores)$,
$deliver(Detrivores, CO_2, Herbivores)$,
$deliver(Carnivores, Carnivores, Detrivores)$,
$deliver(Herbivores, Herbivores, Detrivores)$,
$deliver(Detrivores, Nutrients, Plants)$,
$deliver(Detrivores, Water, Plants)$,
$deliver(Detrivores, Water, Plants)$,
$deliver(Detrivores, O_2, Plants)$,
$deliver(Detrivores, CO_2, Plants)$,
$deliver(Plants, Plants, Detrivores)\}$
and *ReceiveBehaviors* are the corresponding *reception behaviors*;
- which are the *group elementary economic behaviors*;
• $GE2Exch^t = \{ge2exch_1, ge2exch_2, ge2exch_3, ge2exch_4\}$
- which are the *group elementary economic exchanges*;
• $GE2Proc^t = \{e2proc^t\}$
- which is the only *group elementary economic process*.

Given the above *elementary economic model* (EES^t_{EcoSys}), it should be possible to determine, for any given ecosystem (*EcoSys*) that complies with the general model of ecosystems shown in Figure 6, if it is, at any given time, in *material* or in the *operational equilibrium*, whenever the required values of *temporal rates of behaviors* and *material outcomes of exchanges* are provided.

Notice that, in EES^t_{EcoSys}, we have determined all reinforcement signals to be positive for the sake of the example. In any concrete analysis, the correct signals should be appropriately determined.

11. Conclusion

The concepts introduced in the present paper seem to be the most elementary economical concepts that can fit the basic features of energy systems of material agent societies, as they were defined in [1].

The definition of *full-fledged economic systems* for material agent societies is reserved for future work. It will require the lifting of many of the constraints that are intrinsic to the elementary systems. For instance:

- allowing for *more than two* types of objects in economic exchanges;
- accounting for *indirect* operant conditionings, through chains of indirect partners;
- removing the impediment of *cycles* in economic processes, leading to complex networks of arbitrarily interlinked economic processes and allowing for behaviors capable of *indirect self-reinforcements*.

We remark that we have no knowledge of other reports dealing with the concepts introduced here, analogously to what happened in [1].

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